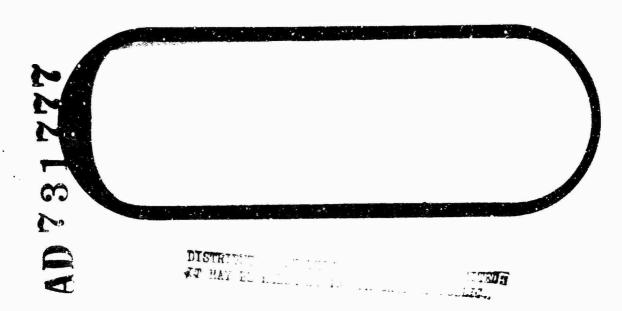
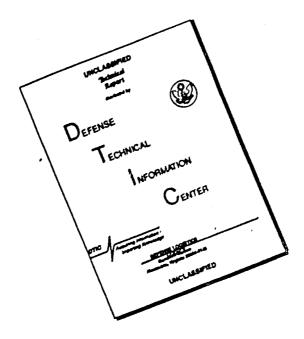
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IMPEDANCE OF ASYMMETRICALLY-FED UNCLASSIFIED TITLE TRAILING-WIRE ANTENNAS KC-135 MODEL _ _ CONTRACT NO. ISSUE NO. __ ISSUED TO CLASSIFIED TITLE ISTATE CLASSIFICATION

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SUMMARY

This paper cansiders the impedance of long wires towed by aircraft for law-frequency communications. Experiments using a small helicopter towing wires up to 1850 feet in length are described. The effects of wire slenderness factor (L/a) and r-f ormic resistance on efficiency and input inpedance are considered. Particular attention is devoted to the synthesis of a model which can be used to predict the impedance of trailing wires for any wire length and aircraft size. It is concluded that wires several miles long towed by aircraft need not be excited at their centers, but can be conveniently and efficiently driven from the aircraft.

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IMPEDANCE OF ASYMMETRICALLY-FED

TRAILING-WIRE ANTENNAS

NOT REPRODUCIBLE

Introduction

There is currently considerable interest concerning the radiation of LF electromagnetic signals from et aircraft. Of particular importance is the impedance of long trailing wires (TW) towed behind the aircraft, and the dependence of wire impedance on wire ength and frequency. If this essay a simple model for the impedance of TW antenna's is presented. Predictions based on this model are then computed with scale-mode measurements obtained from helicopter experiments. Thus validated, the model is used to predict the impedance of very long TW antennas such as might be employed in long-haul VLF communications from jet aircraft.

The long trailing wire antenna excited asymmetrically at the aircraft can be analyzed by asymmetric dipole theory, or by its mirror-cousin, sleeve-dipole theory. The subject has been considered by Tai¹, King², and Taylor³, and has received continued attention by workers at Stanfo d Research Institute⁴⁻⁹.

In the simplest model for the gircraft antenna combination, the aircraft and wire impedances are referenced to a flat imageplane located just behind the aircraft. An image plane everywhere normal to electricfield lines does not distort these lines and thus does not upset the TW impedance. While this stratagem neatly resolves the TW impedance into two easily verified measurables, Co and Zw, one may well doubt whether there is any equipotential surface behind the plane which is flat enough to be replaced by a metal image plane without disturbing the r-f fields. Even if there were such an equipotential surface, there is no apriori knowledge of its location with respect to the aircraft.

Despite these reservations, the simplified flat-plane model should yield a fair approximation for the TW impedance which could be tested by experiment.



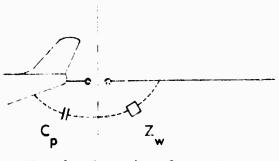


Fig. 1 Equivalent Circuit for Assymmetrically-Fed TW Antenna

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Aircraft Capacitance (C_p)

At frequencies of interest, the aircraft is small in wavelengths and can be represented by a lumped capacitor, C_p . The aircraft must be oriented with respect to the plane such that

the field lines are disposed similarly to the TW configuration. This is brought about by making the ground-plane normal to the TW (dotted line in fig. 2), as the wire unfurls almost straight behind the plane (KC-135) in flight.

in model work, a helicopter was used to support a TW vertically. The aircraft is thus oriented with respect to the ground-plane as shown. The static capacitance problem has been treated in detail in another paper (Appendix A) and the significant results are given below.

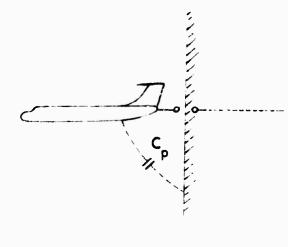
Capacitance to	Capacitance to
Free-Space (pf)	Groundplane (pf)

		•
707	1100	1800
Bell 47-G	200	480

Wire Impedance (Z_w)

The impedance of a long-wire has been treated by Schelkunoff⁽¹⁰⁾ for cases where the stenderness ratio (\mathcal{L}/a) of the wire runs up to 10^4 .

in the present invenstigation very thin wires of $L/a \sim 10^7$ became of interest. Inasmuch as the input resistance and reactance of thin wires changes rather slowly with the logarithm of L/a, it is probably defensible to extrapolate Schelkunoff's data to higher L/a ratios (Fig. 4). Figures 5 and 6 give input resistance and reactance of thin wires for various L/a ratios. Note that lossless conductors are assumed; in the practical case the wire losses will also rise with L/a. This problem is treated in a later section.



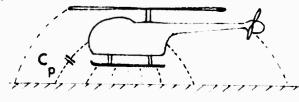


Fig. 2 Aircraft Capacitance, Cp

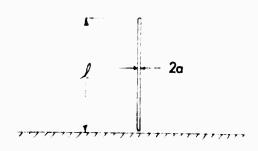
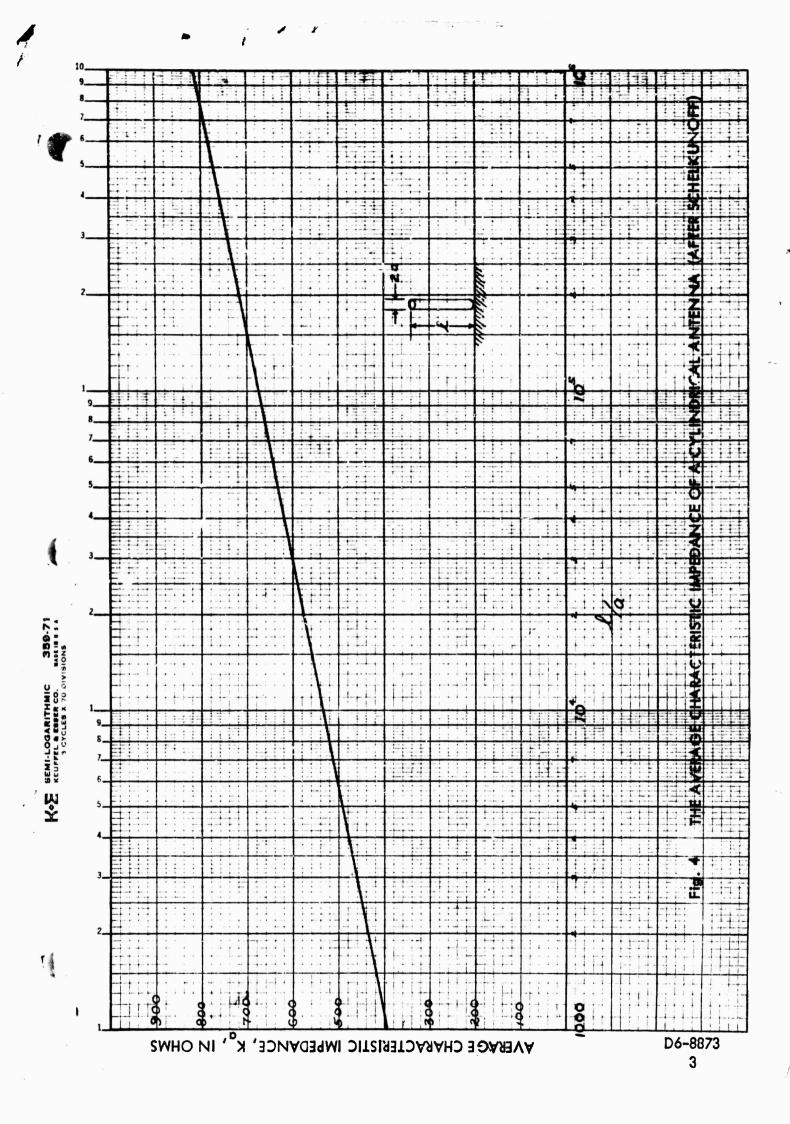
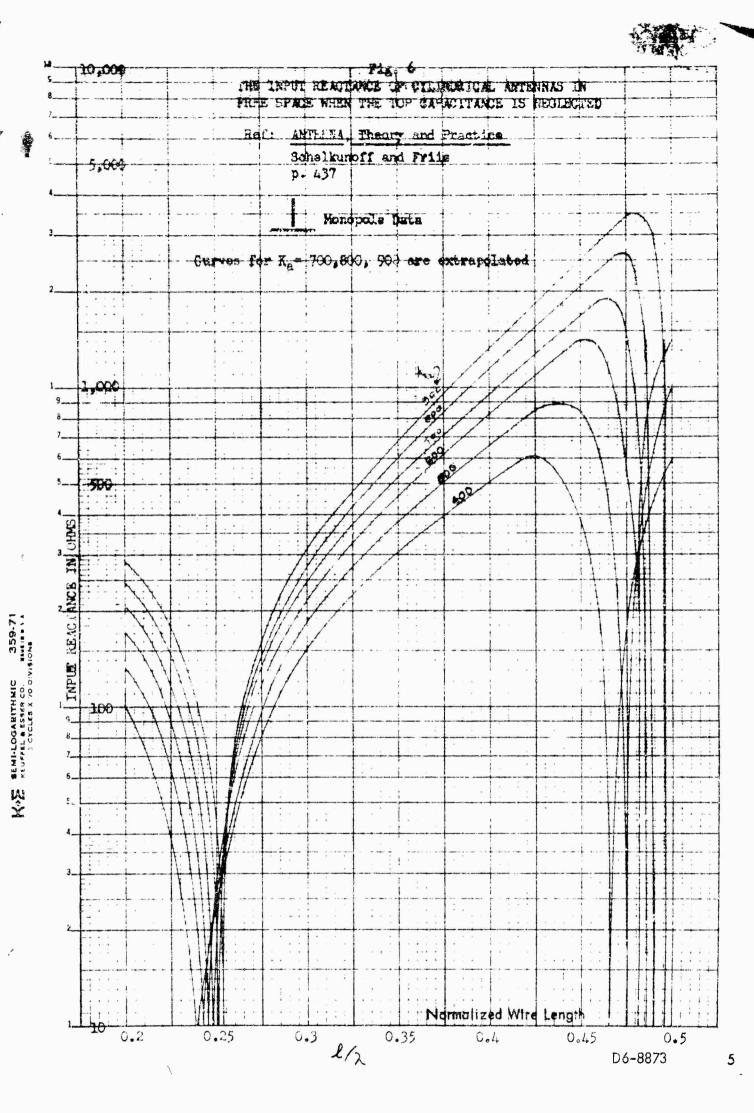


Fig. 3 Wire-Impedance Model

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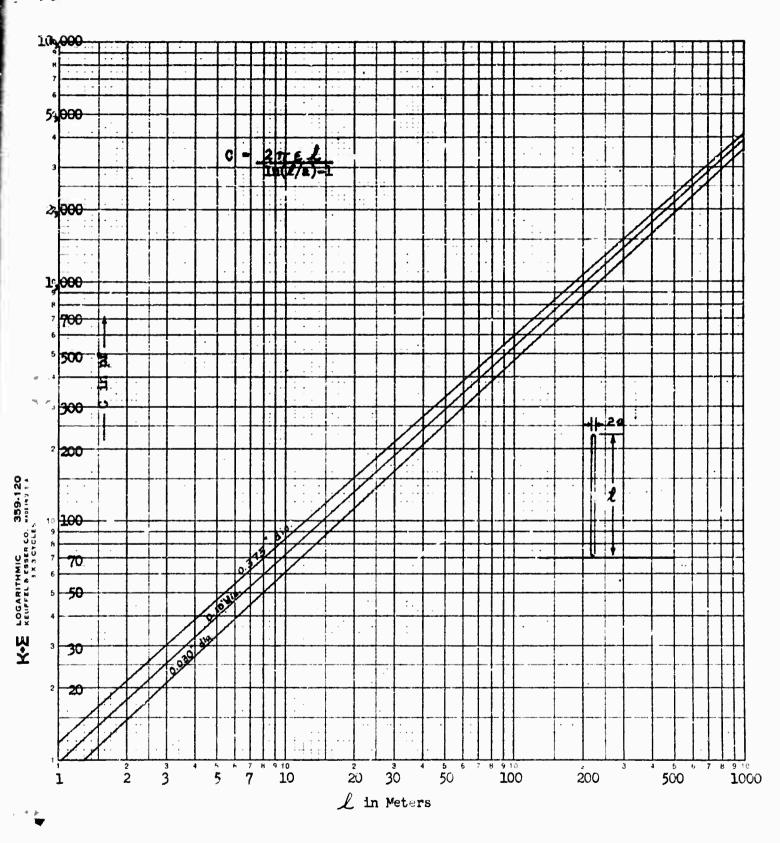


Fig. 7 CA. ACITAMCE OF SLENDER MOLOPOLES ABOVE GROUND

At low frequencies the wire radiation resistance drops toward zero, and the reactance reduces to the quasistatic value given by

$$X = \frac{1}{\omega C_o}$$

where C_o is given by (Fig. 7)

$$C_{o} = \frac{2 \pi \epsilon \mathcal{L}}{\ln \left(\frac{\hat{\mathcal{L}}}{a}\right) - 1}$$
 (MKS)

Impedance of Lossy TW Antennas

The characteristic Impedance, K_{α} , of long, slender antennas has been considered by Schelkunoff (16). In his analysis, the antenna is considered as a transmission line loaded at the far end by an impedance representing the radiation losses. The characteristic impedance for cylindrical antennas is given in Fig. 4.



Fig. 8 Schelkunoff's Transmission Line Equivalent of a Slender Dipole

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Scheikunoff's work can be extended to include lossy antenna conductors using the scattering-coefficient method (11). The underlying idea is that as the attenuation of a four-terminal network increases, its input-impedance spiral collapses about Z_0 , the characteristic impedance. The network efficiency can be obtained by measuring R, the radius of the Z-circle on the Smith Chart (Fig. 7). In the present problem we compute efficiency first and then draw the appropriate Z-circle to derive the impedances.

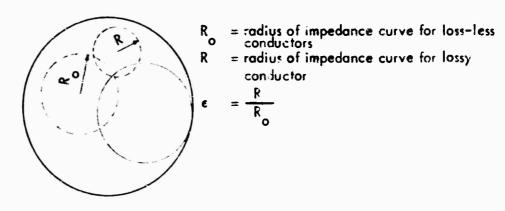


Fig. 9 Graphical Method for Determining Attenuation by Impedance Measurements

The efficiency of long trailing wires operated near half-wave resonance was derived (Appendix B):

$$\epsilon = \frac{(\frac{336}{\lambda})}{(\frac{336}{\lambda}) + R_{\text{inf}}}$$

where λ is the operating wavelength in meters and R_{hf} is the wire r-f resistance in ohms/meter. R_{hf} is greater than the d-c resistance, due to skin effect, and must be determined by measurement or calculation.

The procedure for determining the input impedance of a lassy TW antenna is to:

- (1) draw the Z-curve of the lossless TW anterina on a Smith Chart, normalized about the Z_o obtained from Fig. 4,
- (2) determine the radius R of the lossy Z-circle from the efficiency, ϵ , and from the relation

$$\epsilon = \frac{R}{R_o}$$

where R is the radius of the lossless Z-curve

- (3) draw the lossy Z-curve using a compass, but flaring out at the 'ow frequency end to meet the lossless Z curve (Fig. 10)
- (4) For very lossy wires the characteristic impedance is capacitive; this is accounted for by off-centering the compass when drawing the lossy Z-curve. The lossy characteristic impedance is approximately (for negligible conductance losses)

$$Z_{o}' = \sqrt{\frac{R_{hf} + i\omega L}{LC}}$$

$$= \sqrt{\frac{L}{C} - i\frac{R}{\omega C}} \frac{L}{L}$$

$$= \sqrt{Z_{o}^{2} - i\frac{R}{\omega L}} Z_{o}^{2}$$

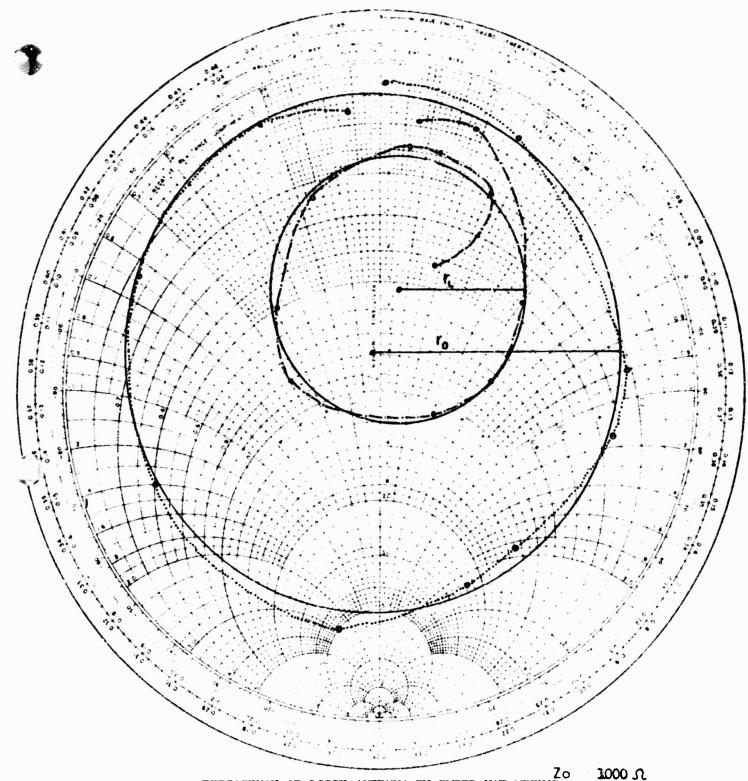
$$= Z_{o} \sqrt{1 - \frac{i}{Q}}$$

$$(4)$$

where Q is the quality factor of a short straight sample of the antenna wire. Measured Q-curves for several typical wires are given in Fig. 11.

Results for several lossy wire monopoles which were measured (Fig. 12) at a scale frequency of 150 mc are tabulated in Table I. No claim for great accuracy is made, but the method is simple, gives \pm 20% accuracy and is intuitively satisfying.

IMPEDANCE-SMITH CHART PLOT



EFFICIENCY OF LOSSY ANTENNA BY IMPEDANCE METHOD

ВО	EING AIRPLAN	E COMPANY	
ANTENNA TYPE Half-wave Nonop	ole	MOCKUP Nichrome 0.001 dia	
FUNCTION	3	MODEL SCAL (2 strands) MODEL FREQ	150 mc
RPLANE		REMARKS	
SPEC. REQ. FREQ. MC-S MEASURED BY Sikorski	VSWR < 11 % DATE 2-62 DATE 2-62	FIGURE NO TEST REPORT	PAGE
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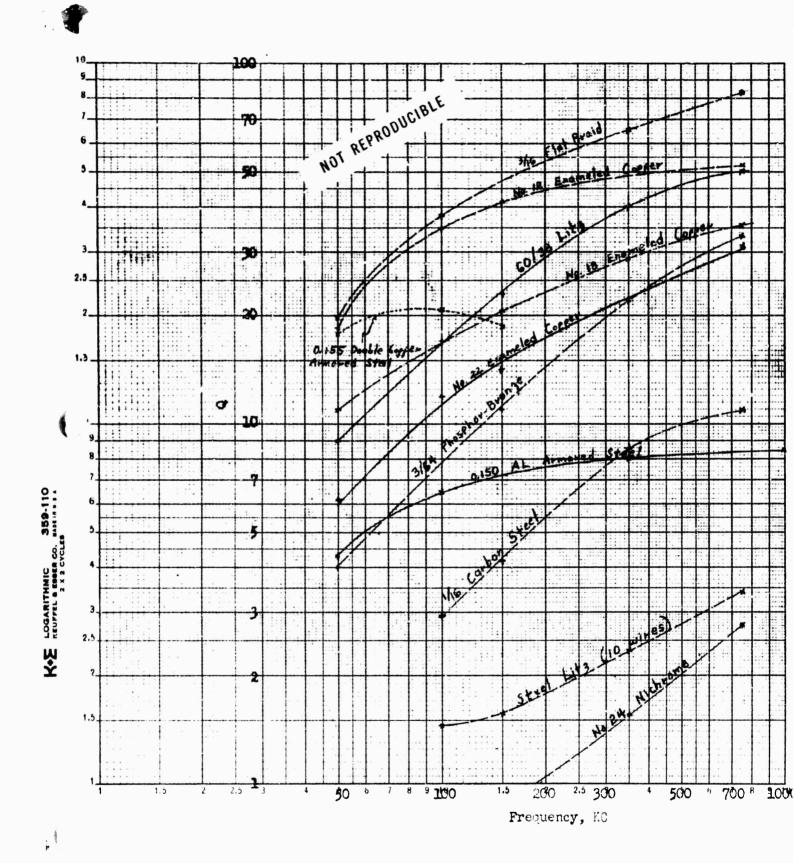
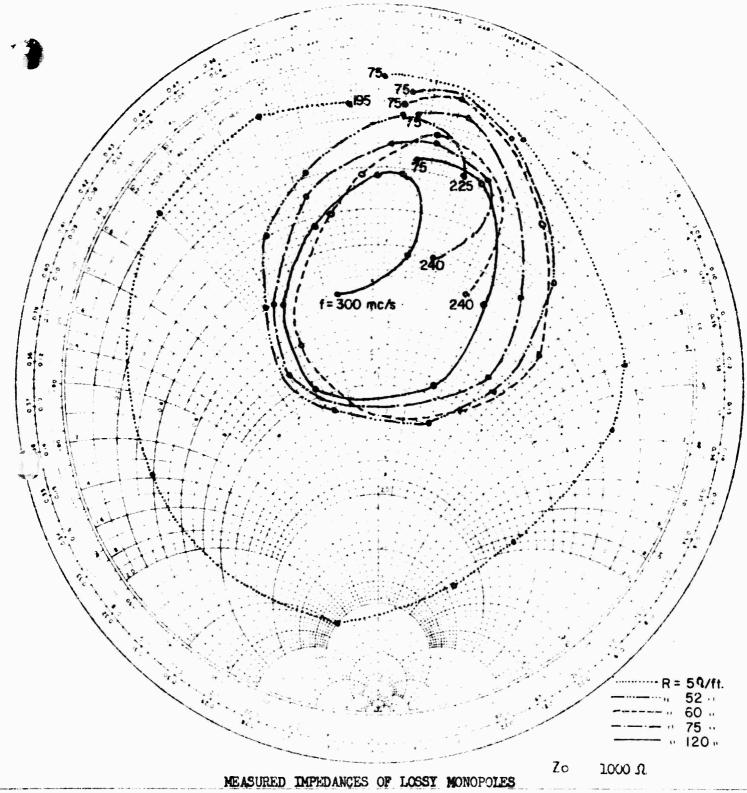


Fig. 11 MEASTLED UNLOAPUD Q OF VALCTOUS LITHES

IMPEDANCE-SMITH CHART PLOT



BOEING AIRPLANE COMPANY

	ANTENNA TYPE	Half-wave	Monopole			MOTHER	Nichrome	0.001 ⁿ		
y '	FUNCTION RPLANE COCATION					MODEL SE	Act 1-5 stra	MODEL ands	FREQ :	150 mc
	SPEC. REQ. FREQ. MEASURED BY C. CHECKED BY J.		DATE	< 3-62 3-62) <u>.</u> - I	FIGURE NO	-	REPORT		PAGE
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R _{hf}	Radius at 150 mc	Method I (Fig.10)	Method II (Appendix B)			
0 (perfect conductor)	3.0 inch (est)	1.0	100%			
5 A./ft	2.9	97%	9 2%			
52	1.5		52			
6 C	1.5 }	50%	48			

Efficiency

37%

43

32

Table I. Comparison of Two Methods for Computing the Efficiency of Lossy Monopoles

1.1

Helicopter Experiments

75

120

It is customary to use scale models in aircraft antenna development work. However, a 1/10 or 1/25 model of a KC-135 towing say a 2000-ft wire cannot conveniently be operated sufficiently high above ground to avoid earth conduction effects. The effect of the earth is not easily predictable from the scanty literature available. Recent work (12) has shown that ground losses acception raise or lower the resistance of low antennas, but it is better to avoid ground effects altogether. Difficulties also arise in attempting to scale the wire diameter, as the wire conductivity cannot easily be scaled upwards, and scaling $R_{\rm hf}$ by proper choice of wire diameter leads to incorrect scaling of the L/a ratio. To avoid ground effects we choose a helicopter - supported wire; to reduce scaling errors we use a low scale factor. Based on capacitance measurements (Appendix A), the real copter scale factor is considered to be 5.5 ± 0.5 when scaling up to the KC-135.

The helicopter was a two-place Bell 47-G (Figs. 13, 14). The antenna winch and impedance operator's position was set up for maximum ease of operation and to keep r-f leads short. The wire dropped straight below the winch through a hole in the cabin floor, minimizing base capacitance.

Wires were cut to exact length and fied to the winch with hylon lanyard. This avoided the necessity for insulating the winch or drum, and minimized base capacitance.

Total base capacitance down to point P was less than 10 pf and can be ignored in analyzing the results of the impedance measurements.

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Fig. 14

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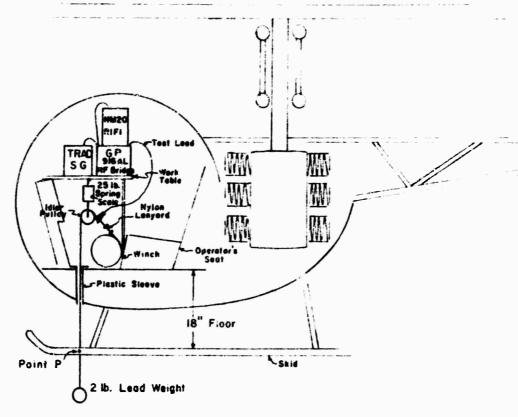


Fig. 15 Outline of Helicopter-Winch Setup

Results

Measured impedance of several 500-ft wires is shown in Fig. 16-18. The measured (in the lab) loss resistance of these wires, Rhf, is noted on each figure. Measured data for a 580-ft litz wire is given in Fig. 19. Measured data for several 1000-ft wires is given in Fig. 20-23. Similar data for the longest wire measured to date, 1850 feet, is given in Fig. 24.

Empirical Model

The simple analysis presented in the introduction only approximates the actual situation. The tiny aircraft is imperfectly coupled to the long antenna. Based on the helicopter measurements, an empirical model for TW impedances is postulated:

- 1. A coupling capacitor, C_c
- 2. An ideal transformer, I:N
- 3. The wire impedance, $Z_{\mathbf{w}}$ (Schelkunoff and Friis)

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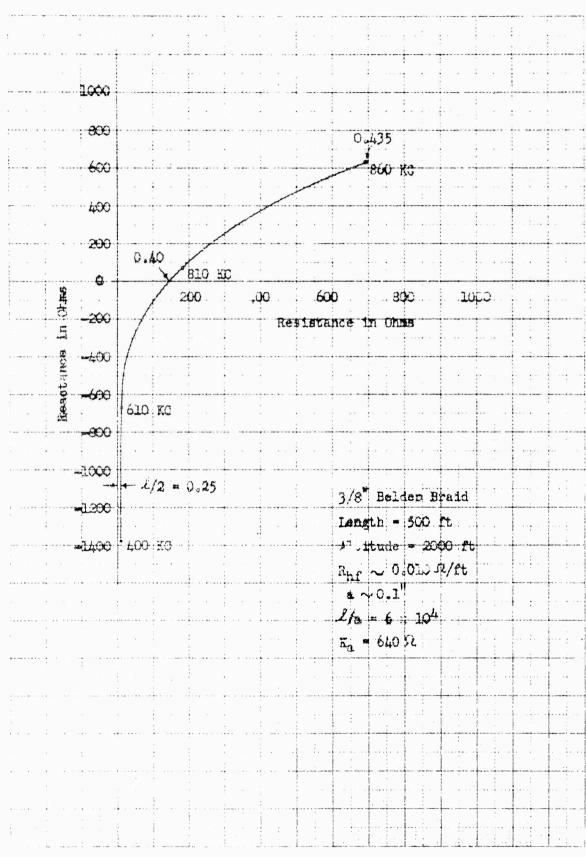


Fig. 3.6 In the live of lead 47-G bullingers. The product 500-Form very

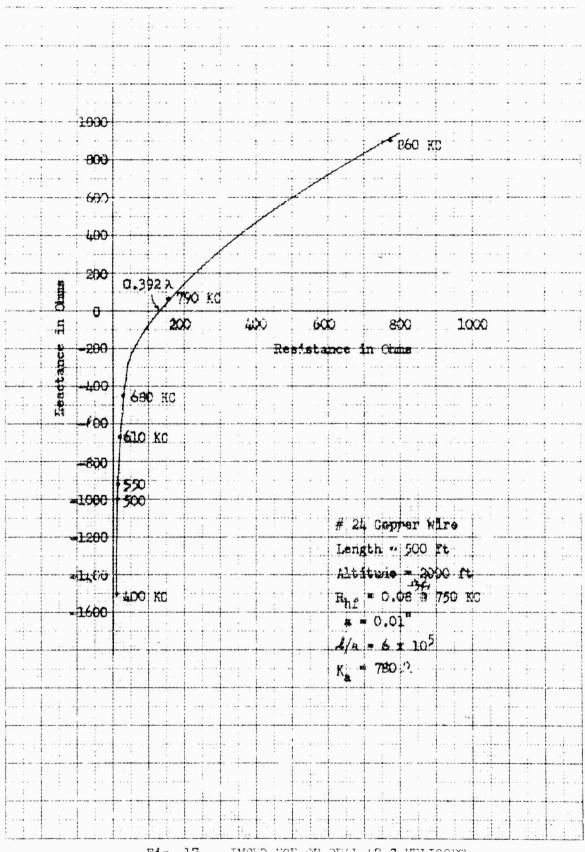


Fig. 17 IMPVD NOE OF BELL 47-3 HELICOPTA ROUNN 4 500-POOT WIRE

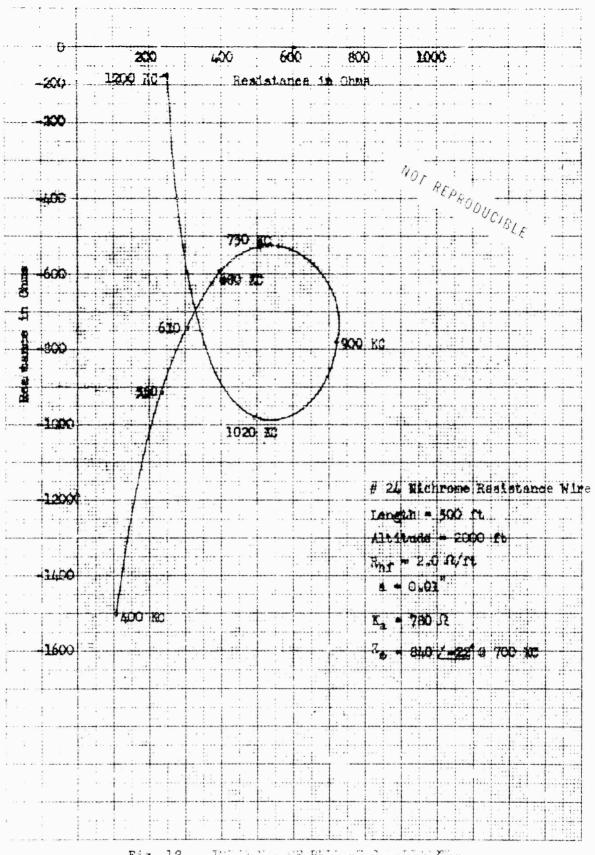


Fig. 18 INTO THE OF BELL AND STORE MINE OF WING A 500-ROOT MIGHT MINE

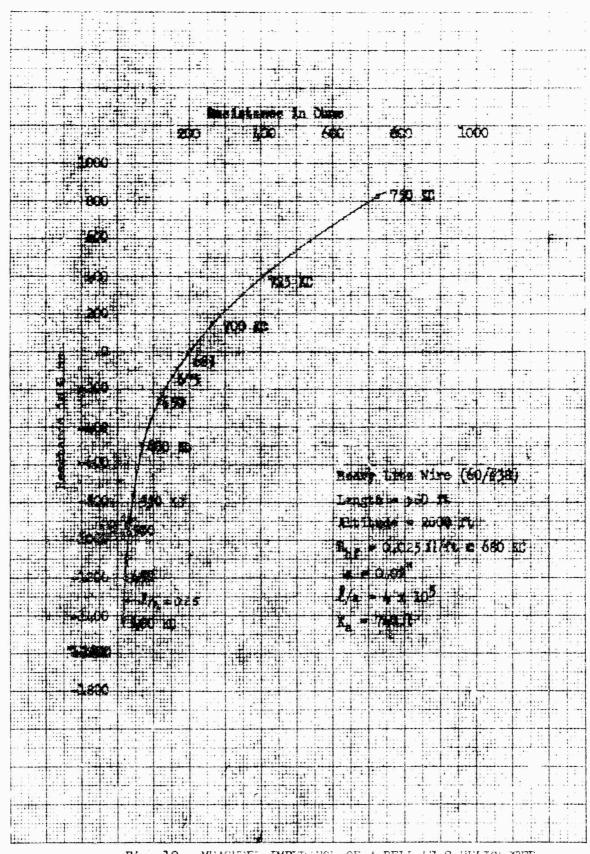


Fig. 19 MEASURED IMPEDANCE OF A BELL 47-G HELICOPTER TOWING A 580-FOOT LITZ WILL

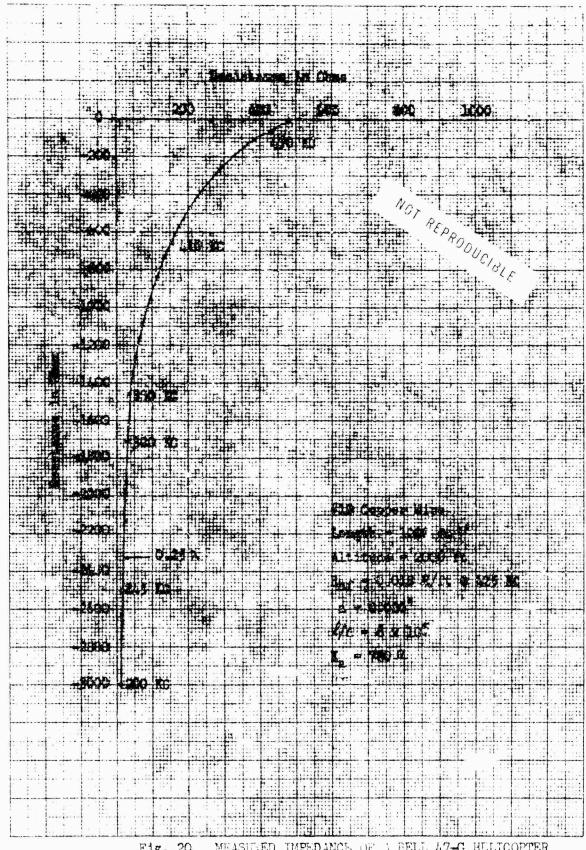


Fig. 20 MEASURED IMPEDANCE OF A BELL 47-G HULICOPTER
TOWING A 1000-FOOT COPPER WIRE

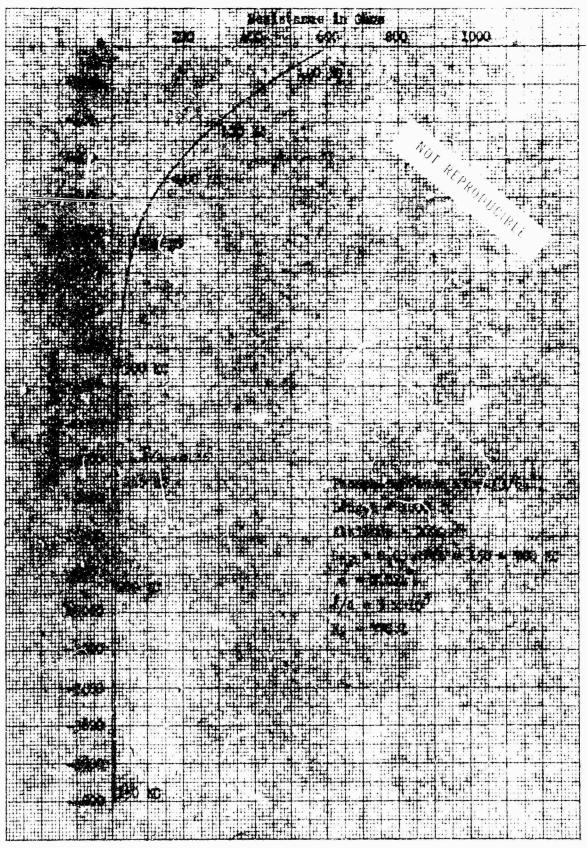


Fig. 21 MEASURED IMPEDANCE OF A BELL 47-G HELICOPTER
TOWING A 1000-FOOT PHOSPHOR-BRONZE WIRE

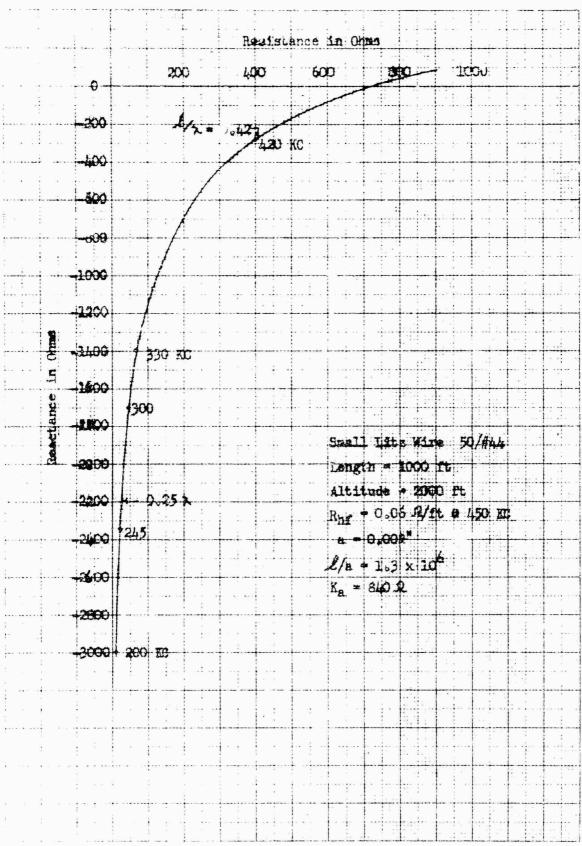
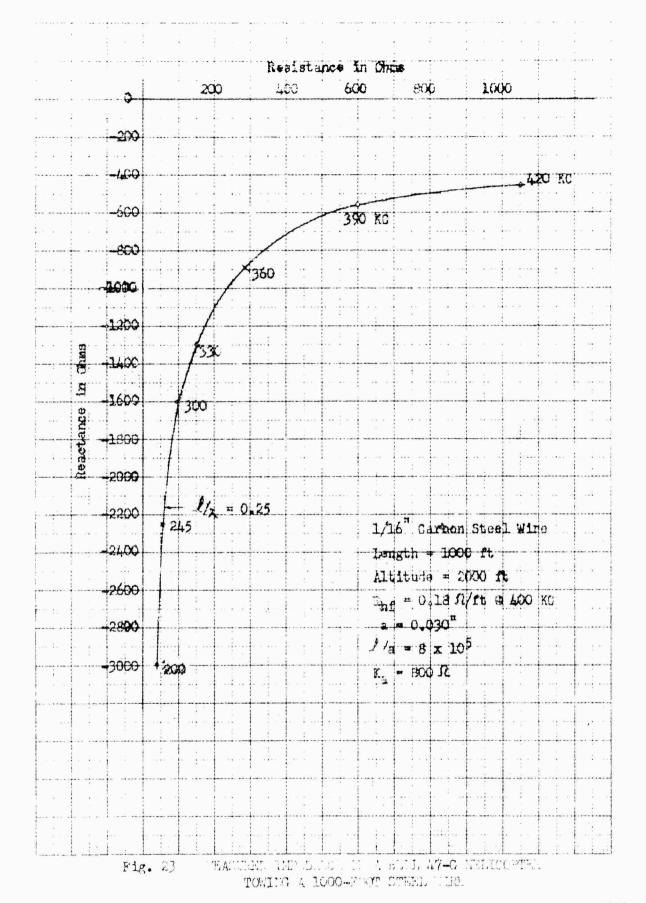


Fig. 22 UHASCLED IMPEDANCE AS A BELL 47-G HELICOPTER TOWING A 1000-POOT LITZ WIRE



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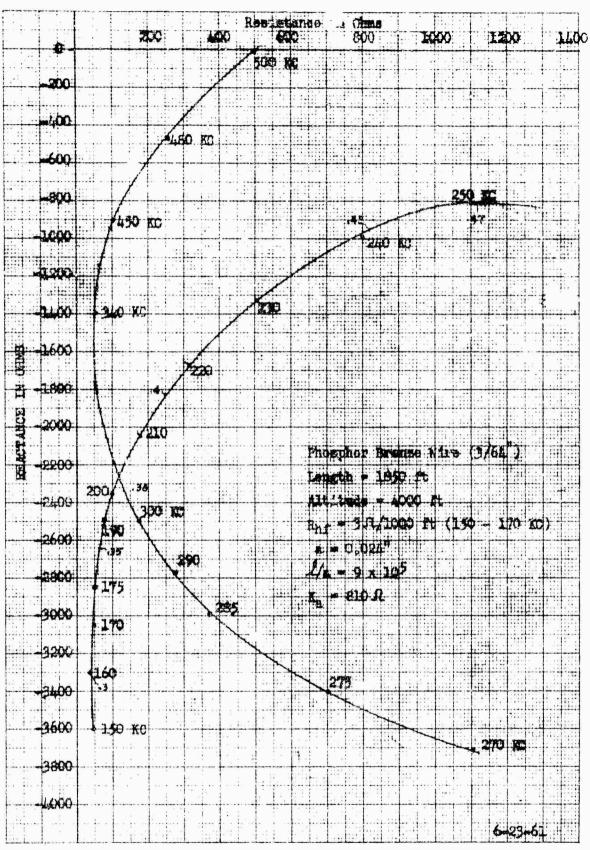


Fig. 24 MEASURED INPERIANCE OF BELL 47-G HELICOPTER TOWNS IN 1850-FOOT PHOSPHOR-BRONZE WIRE

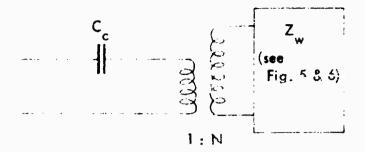


Fig. 25 Equivalent Circuit of Trailing Wire Antenna

The synthesis procedes as follows:

- (1) The coupling copacitance, C₁, is estimated from the free-space capacitince and from ground-pline measurements is in Appendix A₂, C₁ is roughly the mean of the free-space capacitance and the cipacitance over a ground-plane.
- (2) The transformer ratio is probably a function of aircraft size and wire length. For the KC-135 in the 40-150 kc region their tio is

(3) The wire impedance, $Z_{\overline{w}}$, is taken from the appropriate R and X curves of Figs. 5 and 6, and connected across the output of the ideal transformer as in Fig. 25.

To test the validity of the model, we examine the impedance spiral at several check-points: the quasi-static or 'd-c' capacitance, quarter-wave resonance, and 'cross-over', the zero-reactance point.

Quasi-static impedance: At d-c, Z_w reduces to C_w given y (1). Carrying C_w thru the transformer, the equivalent circuit X_w Decomes

$$\frac{C_{c}}{C_{w}N^{2}} = \frac{2\pi\epsilon l N^{2}}{\ln(l/a) - 1}$$

Fig. 26 Equivalent "DC" Circuit of a TW Antenna

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The quasi-static capacitance as a function of wire length is given in Fig. 27 for the Bell 47-G relicopter.

Quarter-wave wire resonance: the wire impedance Z_w reduces to $R_w = 35\Omega$ (no reactance) and the total antenna reactance reduces to that given by $X = \frac{1}{\omega C_c}$. The point $\ell/\lambda = .24$ or .25 ir marked on Figs. 16, 22 and 23.

The equivalent circuit is then

AND ASK DAY

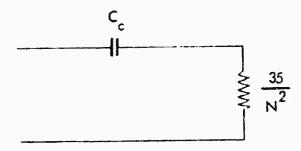
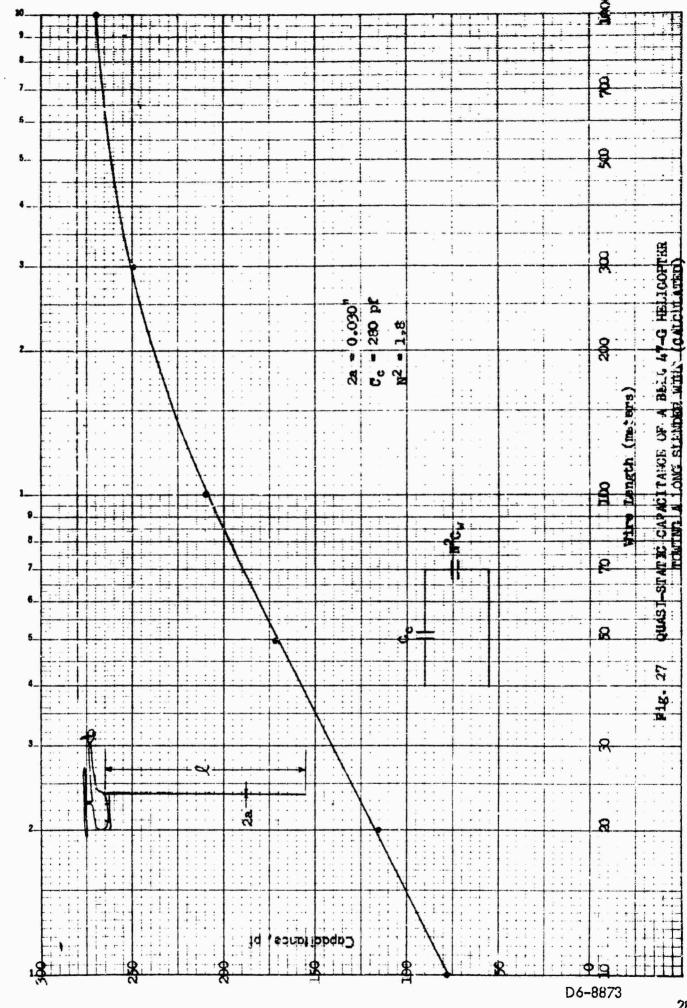


Fig. 28 Equivalent Circuit of TW Antenna at $\ell/\lambda = .25$

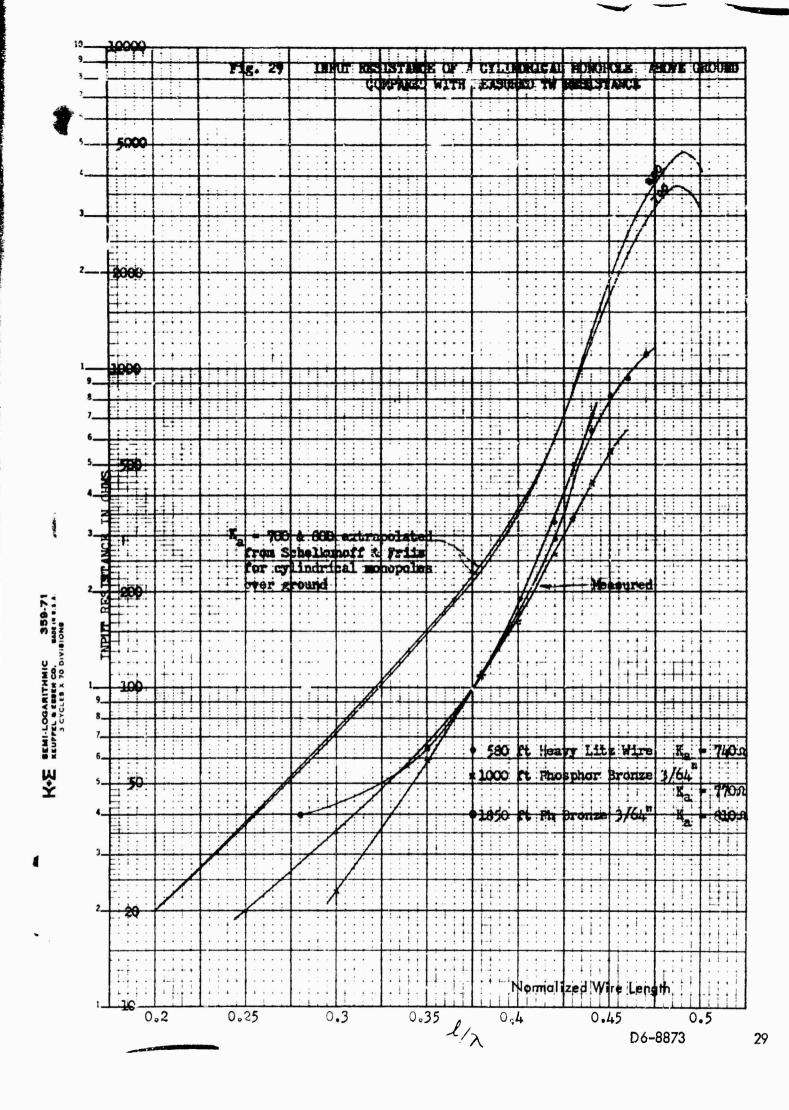
"Cross-over" is where the wire inductive reactance just cancels the capacitive reactance of C. With the longer wires used at the lower frequencies, the wire inductive reactance is insufficient to cancel C, and the antenna impedance spiral remains in the capacitive quadrant.

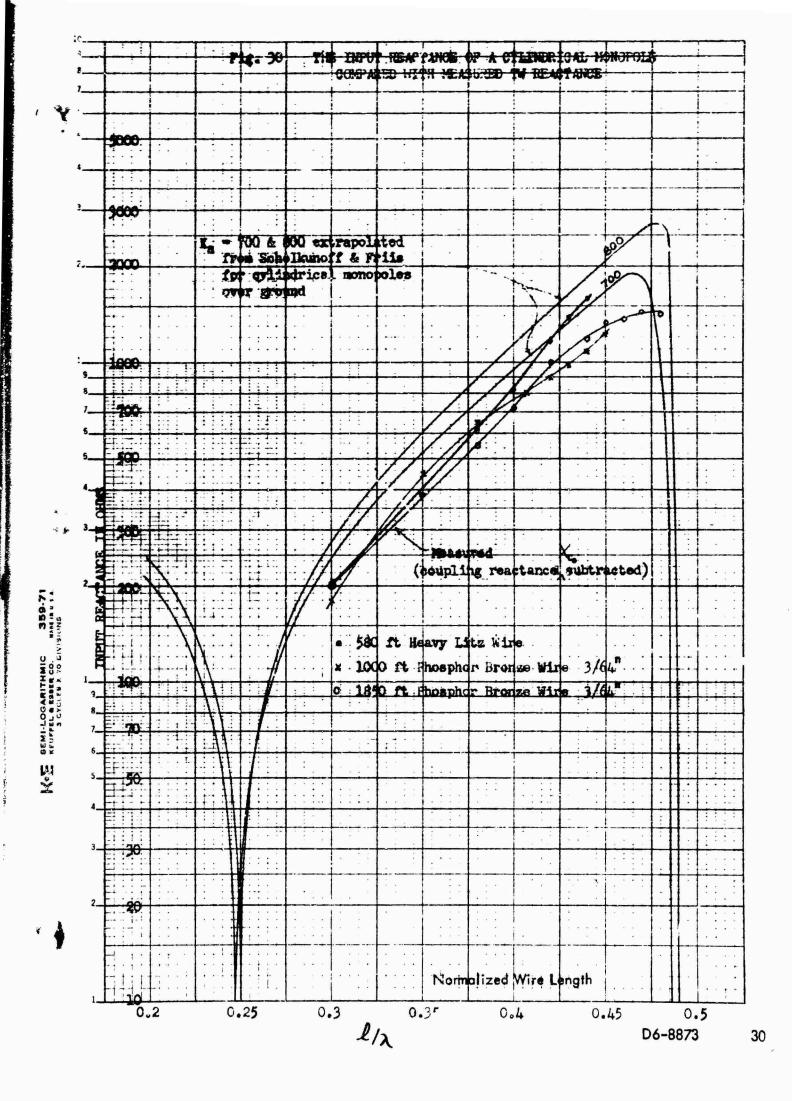
At wire lengths were $\frac{1}{\lambda} > .35$, the wire resistance rises rapidly and begins to exceed the reactance (i.e., the apparent Q drops). In this region the calculated resistance is very sensitive to the choice of an empirical model. Antenna resistance as a function of length (ℓ/λ) is given in Fig. 29. The difference between measured and calculated resistance gives the transformer ratio, N2, of Fig. 25. Similarly, calculated and measured reactances are given in Fig. 30. The transformer ratio, $1/N^2$, represents the difference between the finite aircraft and a perfect, infinite counterpoise. This ratio does not cause a real loss of power, but does lead to higher antenna Q's than if the same wire were fed from the ground,

The coupling factor $(1/N^2)$ from Figs. 29 and 30 is about 0.56. Intuitively, one would expect lower coupling factors for the longer wires; however, in this effect occurs it is obscured by measurement errors, the uncertainties in extrapolating the K curves of Schelkunoff and Fries, and wire losses.



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Wire Losses

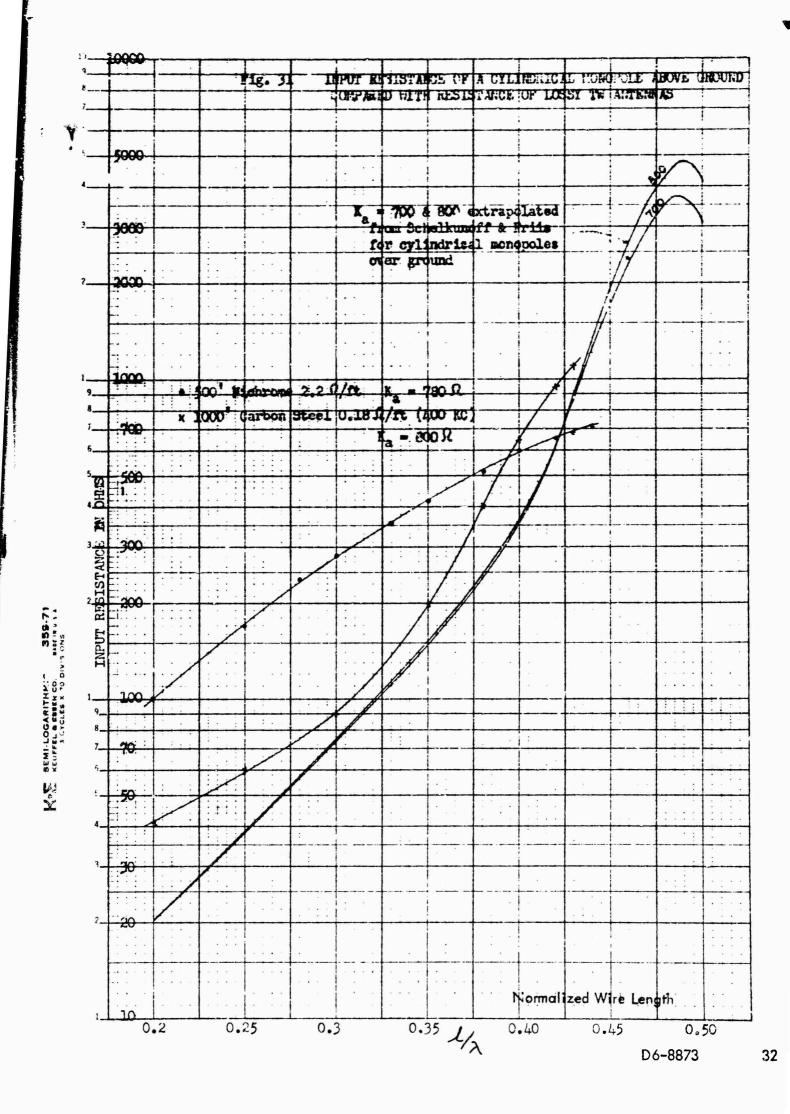
The effect of wire lasses is to callapse the antenna impedance spiral. Wire lasses also affect the characteristic impedance, and in a quite complex fishion as most wires likely to be used (diameter .02 - .16 inch) puss from a do-current carrying to a skin-effect regime in the frequency band of interest (100-1000 kc)¹³. Resistance and reactance curves for several lossy antenna wires the given in Figs. 31 and 32. Note that increasing losses raise the antenna resistance at low frequencies but lower it at higher frequencies.

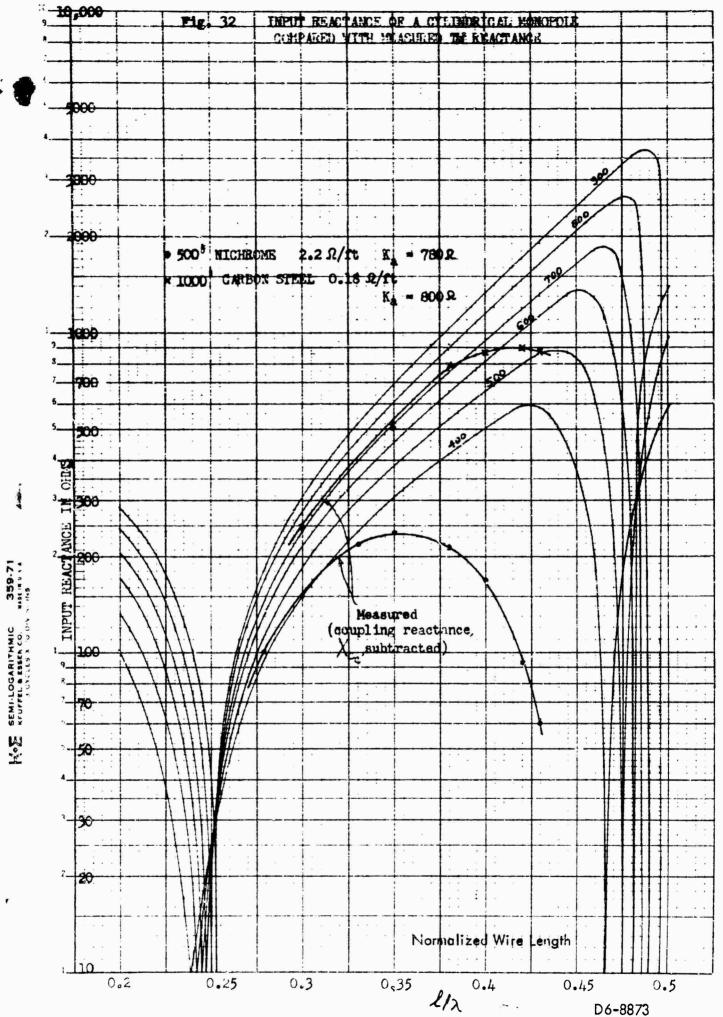
Dunking Measurements

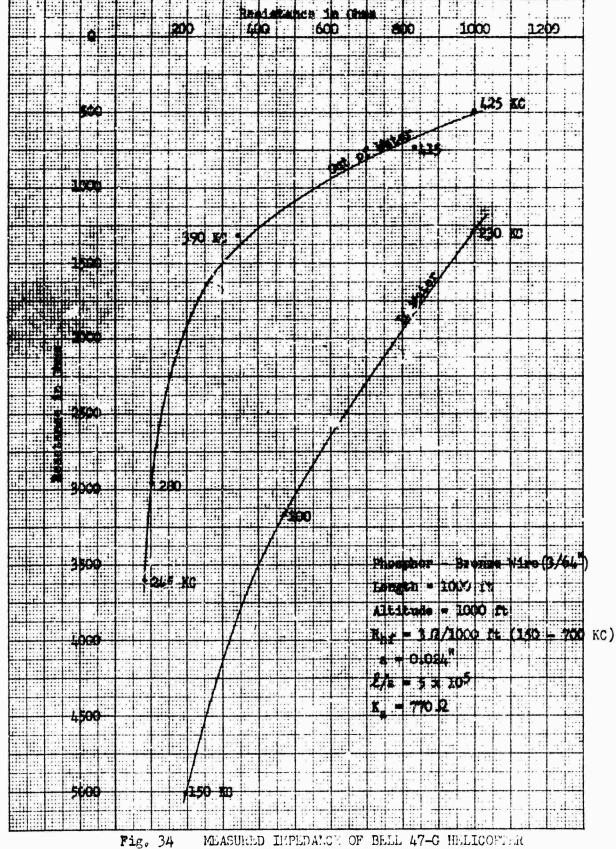
Graunded-manapole measu ements were made using a wire driven from a nelicopter. A 1000-foot and a 1850-foot wire was suspended over Puget Sound with the lower and dunked into saltwater. A shiny gallon can poked with hales was suspended at the lower end. The can was asserved visually from the helicapter to indicate contact with saltwater. The can also increased the surface area of the wetted end of the wire, reducing the 'ground' resistance of the antenna to a negligible value (Appendix F). Fig. 34 shows the impedance of a 1000-ft 3/64" phosphor-cronze wire suspended from a elicopter in both dunked and free-space conditions. Similar data for a 1850-ft wire is shown in Fig. 35. As might be expected, grounding a trailing wire in saltwater lowers the operating frequency by approximately one half without lengthening the wire. The mirror-image of the wire in the saltwater pravides a classic example of the doubling of the effective length of the wire. Noting that LF propagation over salt water is much better than over land, the nelicopter equipped with a few pounds of wire provides a remarkably effective and mot le platform for LF compunications.



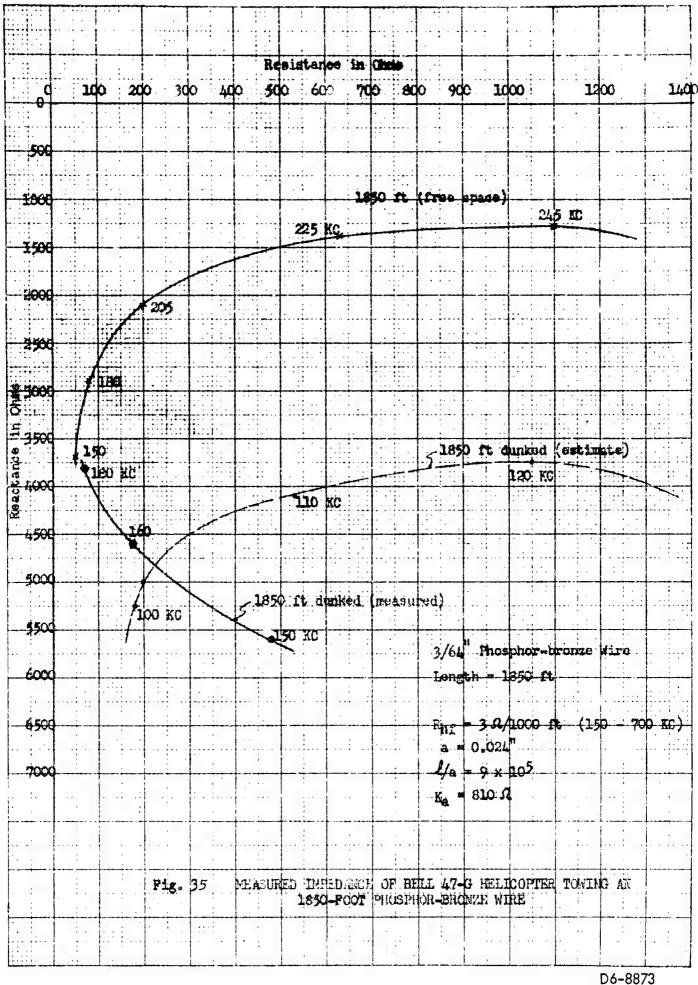
Fig. 33 TW Dunking Test Over Puge: Sound







MEASURED IMPEDANCE OF BELL 47-G HELICOFTER TOWING A 1000-FOOT PHOSPINER-BRONZE WIRE



Conclusions

- 1. A simple model for predicting the impedance of TW antennas has been postulated and verified.
- 2. The effect of ormic losses on the input impedance of TW antennas cannot be neglected and can be predicted fairly well using a method corrowed from scattering-matrix theory. The extension of this four-terminal technique to two-terminal devices such as aerials, rests on tenuous theoretical grounds and requires deeper analytical treatment than our schedule or inclinations permit.
- Ohmic losses in the antenna wire can either raise of lower TW input resistance, depending on the normalized length ℓ .
- 4. The helicopter supports wires vertically, which is a natural position for dunking as well as for launching groundwaves at low frequencies. The large helicopters coming built today (such as Boeing Vertol 107) would appear to make ideal launching platforms for continent-wide 1-f communications. For some applications, the wire could be supported from a helicopter, but excited from the ground (or shipboard).
- 5. Although the empirical model suffices as a first approximation, it can stand improvement for better tracking Letween the analytical data of Schelkunoff and Friis and the measured data. Also, the model's domain of validity can be extended to higher asymmetry ratios to facilitate impedance predictions for longer wires in the VLF region.
- 6. Trailing wires need not be fed at the 73-ohm center point. They may be fed at the aircraft and with reasonable resistances, reactances and operating Q. In the 1-1 band, operating wire lengths must be in the range $\ell/\lambda = 0.40$ ---0.50. The optimum operating point for minimum impedance (low driving voltage) is at $\ell \sim 0.44\lambda$.
- 7. The small helicopter is both inexpensive (160/hour) and convenient to use for trailing-wire antenna studies. And the view is terrific.

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STATIC CAPACITANCE OF THE KC-135 AIRPLANE

A model far the impedance of trailing-wire antennas would be useful in the design of LF and VLF radiators trailed behind the KC-135. One such model has been postulated in which the impedance

of aircraft and wire are referenced to an image plane imagined to be located just behind the towing aircraft. An image plane everywhere normal to electric field lines does not distort these lines, and thus does not upset the TW impedance. While this stratagem neatly resolves the TW impedance into two easily verified measurables, Cp and Zw, one may well doubt whether there is any point behind the towing aircraft where an image plane could, in fact, slipped in without disturbing the r-f fields. Even if there were such an equipotential plane, ther is no apriori knowledge of its location with respect to the aircraft.

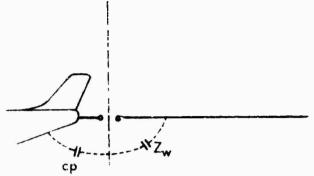


Fig. 1 Idealized Model of TW Antenna

An Alternate Approach

Instead, we consider the law-frequency or quasi-static case where $Z_w = X_w \left(= \frac{1}{\omega C_w} \right)$

and, as before,
$$X_p = \frac{1}{\omega z_p}$$

X_p × X_w

Fig. 2 Quasi-static Model of TW Antenna

The measured capacitance of the driving reminals (Fig. 2) which we call C_{ς} , is then given by

$$\frac{1}{C_s} = \frac{1}{C_p} + \frac{1}{C_w}$$
 (1)

Note that C and C need not be defined relative to a plane, but any convenient equipotential surface will do.

Another capacitance, C_o, represents the free-space capacitance of the aircraft with respect to an infinite sphere surrounding the aircraft. At low frequencies, the aircraft can be replaced by an equivalent sphere, that is a sphere of equal capacitance, Co, given by

$$C_0 = r_0 \text{ (esu)}$$
or
 $C_0 = 1.1 r_0 \text{ pf where } r_0 \text{ is in cm}$
(2)

(1) RE-115, "Impedance of LF Trailing-Wire Antennas"

Calculating C

The free-space capacitance of an irregular body such as the KC-135 cannot be computed exactly. An approximation is considered in ref (2) using ellipsoidal functions. A rough estimate can be made by computing upper and lower bounds. On the high side, if the wings were filled in 'til the airplane resembled a flying saucer, the capacitance of such a disc would be:

$$C_{\text{max}} = \frac{\alpha}{\sqrt{2}}$$
 esu
= 1380 esu
= 1520 pf

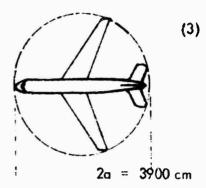


Fig. 3 Disc Equivalent for C max

On the law side, we throw away the wings and consider the fuselage as a prolate spheroid, whose capacitance is:

$$C_{\min} = \sqrt{\frac{\pi}{4}} \quad \text{ab} \quad \text{esu}$$

$$= 545 \quad \text{esu}$$

$$= 600 \quad \text{pf}$$

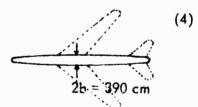


Fig. 4 Prolate Spheroid Equivalent for C_{min}

The capacitance, Co, of the KC-135 is then

A rough estimate would be to average (though truth is seldom midway between two errors!) and we write,

$$C_0 = 1060 \text{ pf} + 30\%$$

(2) RE-167, "Capacitance of Isolated Airplane"

Measurement Techniques

The free-space capacitance, Co, can be measured by numerous methods ranging from d-c charge methods to Q-meter and bridge techniques. All require connecting a lead to the aircraft (or aircraft model) and in each case one must consider the capacitance of the connecting wire, particularly in using small models, where lead capacitance may not be negligible. The measurement of Co begins to resemble C₂. (Fig. 5)

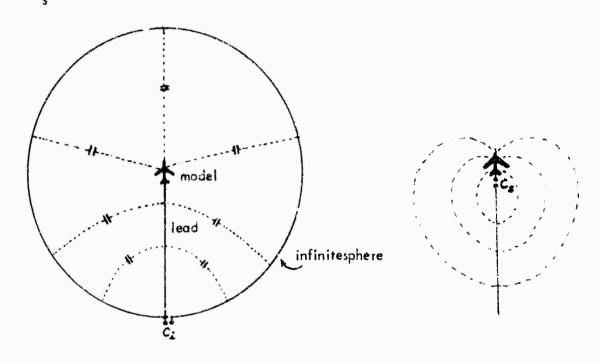


Fig. 5 Measurement of Airplane Capacitance $C_{\underline{a}}$ and $C_{\underline{s}}$.

The capacitance of several KC-135 models (1/120 and 1/25) and of several reference spheres was measured. The 1/120 model was measured indoors where overhead wiring and pipes may have introduced errors. The 1/25 model and sphere were suspended from a 60-ft pole. Various size connecting wires was used. The Q-meter was used to measure capacitance at frequencies low enough to fulfill the quasi-static assumptions (80-120KC). The GR-722 precision variable capacitor was used for all Δ C measurements. To measure C., the capacitance of the feedwire was tuned out with the Q-meter; C. was then taken to be the added capacitance, Δ C, measured when the switch was closed (See Fig. 6).

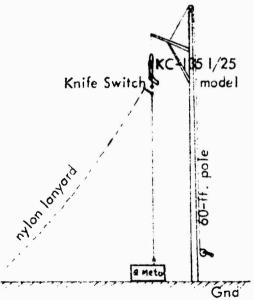


Fig. 6 Setup for Measuring C.

It may be objected that 'ground' isn't really spherical, and isn't infinitely far from the plane anyway. A simple calculation of the capacitance of a spherical concentric capacitor shows that once the radius of the outer capacitor becomes 10 times the inner, the capacitance is essentially C₀. For flat ground, the proximity effect is even less.

C was measured in similar fashion, except that a coax cable was used to feed the model. The internal capacitance of the coax was resonated with the Q-meter. The capacitance of the airplane with respect to the coax shield, C, was taken to be the change in capacitance measured when the switch was closed.

Operating the model several model diameters away from ground insures that most of the electric field lines terminate on the wire, not on the ground. The argument is then that since all field lines from an airplane of radius r terminate on the wire within a distance r or so, the wire could extend to infinity without a significant change in C. Quasi-static results thus may be of help in solving the TW impedance problem for any specified wire length.

ered 25pf ishingly ic to

Fig. 7 Setup for Measuring C

Results

The measured value of C. for 18-inch and 26-inch foil-covered spheres is given Fig. 8. The calculated values of C. were 25pf and 36pf, toward which C. and C. should converge for vanishingly thin wires. For reasons unknown, the measured values were 10 to 20% lower than calculated, despite great care taken to insure accurate results. Evidently there are fringing fields we have not accounted for. Rather than pursue the subject further, we take the sphere data to construct a fudge factor of 1.20 to be applied to the model data. The measured values of C. and C. for the 1/25 scale model KC-135 are given in Fig. 7.

Helicopter Capacitance

Many TW measurements have been made using a Bell G-47 helicopter, and it is of interest to determine helicopter C. and C in order to predict TW impedances for the KC-135. A 1/30 scale metallic model Bell G-47 (Army H 33H), kindly donated by Bell Helicopter Co., was used in a setup similar to that shown in Figs. 6 and 7. The TW drops straight down from a point just in front of the copilofs (passenger) seat just as in the full-scale helicopter TW measurements. C. and C are given in Fig. 10. From this data, the free-space capacitance of the full-size helicopter would be 180 pf. Measurement accuracy is somewhat reduced with such small movels, and the sphere data indicares that our readings are all low by 26%. Till better data is available we take C = 200 pf.

The calculated capacitance of the helicopter, neglecting the wing, is from (4).

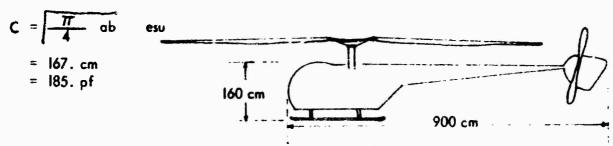


Fig. 11 Helicopter Dimensions

There are two models of blade available, a metal and a wooden version. While Aerocepter's personnel swear that there is no metal in the wooden blade, capacitance measurements (Table 1) clearly show a variation with blade orientation. In the air, prop modulation is clearly noted when near the impedance bridge nulls when measuring TW impedance.

Aircraft Capacitance Close to Ground

The capacitance of a 707 and a Bell G-47 helicopter close to ground were measured. The 707 was sitting on the Boeing flight line and the safety ground was used to connect to 'earth'. The Bell G-47 was perched on a 3-ft stack of wood pallets over an array of 20 18-ft wire radials laid out on the tarmac (Fig. 12). Frequencies below 300 kc were used to avoid transmission line effects. Results are given in Table 1.

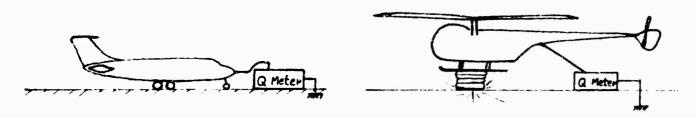
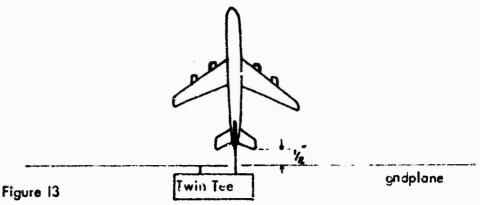


Fig. 12 Full-Scale Aircraft Capacitance Measurement Setup

Aircraft	Capacitance (pf)
707	400C
Bell G-47 wing fore∽aft	480
Wing athwart	500

Table I Aircraft Capacitance Close to Ground

A 1/120 KC-135 model was measured over a small 4' x 4' ground plane (Fig. 13). The capacitance was 1800 pf (corrected to full-scale).



1/120 Scale KC-135 in Tail-Stand over Image Plane

Conclusions

- 1. The static free-space capacitance, C, of the KC-135 is 1100 pf (± 10%); that of the Bell G-47, is 200 pf (± 20%). A factor of 5 1/2:1 may therefore be used to scale up helicopter TW data to predict KC-135 TW impedance.
- 2. For thin wires long compared with aircraft dimensions, but short in wavelengths ($-\frac{9}{2}$ < .1), the capacitance is nearly constant and is about equal to the free-space capacitance. (Lead-in capacitance must obviously be treated separately)
- 3. A method of accurate measurement of C_0 should be devised whereby leads could either be eliminated, or lead capacitance compensated for.
- 4. Conclusions regarding the use of C and C in building an adequate theory of the impedance of highly-asymmetric dipoles must be deferred to a later paper.

C.D. Lunden
Electronics Group
Research Unit
Electrodynamics Staff

CDL:cm

ig. 6 Capacitance of two netal-foll covered spheres

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Fig. W CAPACITANCE OF 1/25 SCAIR IN-135

G-47 HELLICOPTER CAPACITANCE

ON THE L-F RADIATION EFFICIENCY OF TRAILING-WIRE ANTENNAS

It is of interest to determine the losses in long trailing wire antennas used as 1-f radiators on the KC-135 airplane.

At low frequencies (150kc) the skin depth of copper is

$$\int = \frac{1}{\sqrt{\pi f \mu_0}}$$
= 0.16 10⁻³ meter
= 6.6 mil

As F is much lesss than the radius of commonly used trailing-wire antennas, the "h-f" formulas for skin effect may be used. The h-f resistance of a roundwire is

$$R_{h_{f}} = \frac{Rs}{2\pi r} \qquad \text{ohm/meter} \qquad (2)$$

where

$$R_s = \sqrt{\frac{\pi f}{\sigma}}$$
 ohm/square (3)

whence

$$R_{h_f} = \frac{1}{2\pi V_o} \int \frac{\pi f \nu}{\sigma} \quad \text{ohm/meter} \qquad (4)$$

For copper

3

$$R_s = 2.6 \cdot 10^{-7} \sqrt{f}$$

and (2) becomes, at 150 kc

$$R_{h_f} = \frac{1.6 \cdot 10^{-5}}{Y_a}$$
 ohm/meter (5)

The radiation resistance of a resonant dipole carrying uniform current is

- 1. Ramo and Whinnery, Section 6.09
- 2. Kraus, Problem 5-4



The radiation resistance per unit length, R_m, is

$$R_{m} = \frac{336}{2} \quad \text{ohm/meter} \tag{6}$$

We define the antenna efficiency 3 to be

$$\Sigma = \frac{R_{m}}{R_{m} + R_{h_{f}}} \tag{7}$$

For copper wire the efficiency is

$$\Sigma = \frac{336}{336/\lambda + 1.6 \cdot 10^{-5}}$$
 (8)

For a 0.12-inch diameter copper wire at 150 kc, (8) becomes

$$\Sigma = \frac{\frac{336}{2000}}{\frac{336}{2000} + \frac{1.6}{1.5} \frac{10-5}{10-3}}$$

$$(= -0.3 \text{ db})$$

For a steel wire:

Assume:

$$d_0 = 0.12$$
 inch $(Y_0 = 1.5 \text{ mm})$

$$M = 40 \mu_0 \qquad \text{(initial permeability)}$$

$$\sigma = 0.6 \cdot 10^7 \text{ m ho/m} \quad (= \frac{1}{10} - \sigma \text{ copper})$$

from (4)

and

$$\Sigma = \frac{0.168}{0.168 + 0.210}$$
$$= 44\% \qquad (= -3.6 \text{ db})$$

3. Antenna efficiency is to distinguished from pattern efficiency, coupler efficiency or antenna system efficiency.

For aluminum wire:

Assume:

$$d_0 = 0.12$$
 inch
 $\sigma = 3.7 \cdot 10^7$ mho/meter ($\sim .6 \sigma$ copper)

from (4)

$$R_{hf} = 0.013$$
 ohm/meter

and

$$\Sigma = \frac{.168}{.168 + .013}$$
$$= 93\% \qquad (= -0.3 db)$$

The high frequency resistance of wires used as trailing antennas depends on construction and geometric factors as well as material. The actual $R_{h_{\rm f}}$ of various wires to be used in LF and VLF trailing wires can be determined by measurement. A simple test jig for this purpose is now under construction.

C. D. Lunden

CDL:cm cc: Buehler Carman Fairfield Isbell Short

Appendix C

OPERATING NOTES

Inasmuch as operating very long antenna wires from helicopters is a relatively new development, notes on flight experiences may be of interest.

Vehicle

The Bell 47 G-3 is a small, single rotor three-man helicopter with the following characteristics:

Gross oper, weight	2850 lb.
Min. weight empty	1850 lb.
Pay lo. d	1000 lb.
Hover hours	1000 lb. 2 hrs 1000' above sea-level
Ceiling	150001
Gas consumption	18 gal/hr
Power Electric Power Over all length	240 Hp 24v dc 40 amps (continuous) 342"

445"

Operating Areas

Rotor diameter

The FAA is understandably concerned about hazards associated with long dangling wires. Because of lengthy delays associated with the tribal custom of "refering everything to Washington", it was found more expedient to get permission to use airspace controlled by military authorities, which is not under the FAA jurisdiction. A site near Yelm is available on 1-2 day notice from US Army authorities at Ft. Lewis.

Operating over water or sparsely settled areas reduces the likelihood of dropping the wire onto a powerline, or the end-weight through the roof of someone's nouse!

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Appendix C 50

! istallation

The winch, prime power and equipment installation was made to be easily installed or removed in less than one hour. A block diagram is snown in Fig. 1

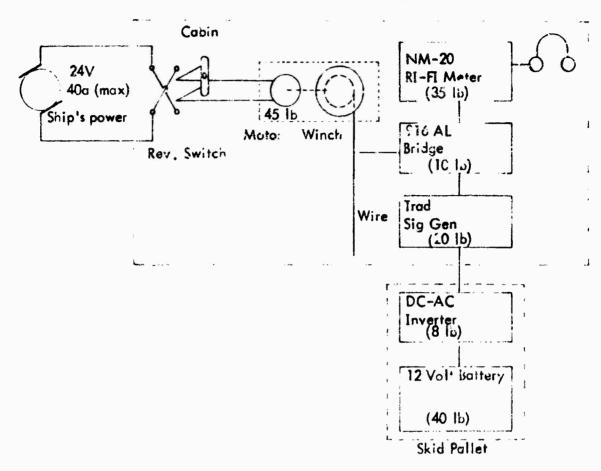


Fig. 1 TW Installation in Bell Helicopter

Operating Procedures and Problems

After installation, the aircraft is flown to the operating site, the wire extended until it runs completely off the winch and is suspended only by a hylon lanyard. This obvious the need for a floating winch, and reduces base capacitance to a minimum. The impedance-trequency run is made, the wire retrieved and the next wire or test begun. For planning purposes, allow one flight hour per R-X diagram.

Hovering presents a problem in aeronautics and psychology. Helicopter lateral control and stability are both poor when hovering in still air. In a 20-knot wind, control and stability improve markedly, and yet a suitably weighted wire still dangles nearly vertically from the helicopter. For hovering in still air (less than 5 knots) a supercharged aircraft may be necessary. For the Bell-47 aircraft, this runs the cost up from around 360 per flight nour to over 100 per flight hour. But the pilot, a skilled and well-paid artist, feels me must be going somewhere to earn his salt. Hovering, gull-like, more than 5 minutes

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over a point is quite out of the question so far as the pilot is concerned. As it turned out, the measurements were made while patrolling Lake Sammamish as it on an ASW exercise.

in-flight Hazerds

NOT REPRODUCIBLE

On retrieval, the wire and end-weight tend to go into spherical pendulum mode of oscillation at wire lengths less than 40 feet. As the wire gets shorter, its oscillations become more extreme (angular momentum being conserved). From observations made at pizza parlors we call this the 'spaghetti-effect'. At wire lengths under 10 feet the wire and end-weight may swing up and hit the helicopter or wisp around the skids. These oscillations can be damped by reaching out of the cabin door with a light 6-ft, crook.

Lightning, while rare in the Seattle area except in midsummer, night conceivably le l'attracted' by long dangling wires. The recent work of Vonnegut (Arthur D. Little Co.) in New Mexico i as terided to dispelithis notion. Sudden injection of a long wire (as by rocket) into an incipient lightning field may precipitate a stroke, but wires carried by aircraft into the same incipient field move much too slowly and are shielded, in effect, by their own space charge.

Charging is well-known to occur on helicopters in flight and was observed on almost every flight in this series of tests. The wire probably enhances the effect by increasing capacitance and hence the charge stared on the system. Never quite getting accustomed to the 'cap' when handling the wire, we soon found that a 10- meganin resistor across the wire to ground (aircraft structure) bled off the charge without introducing errors in the ref bridge reading. There is good evidence that charging burned out the of transformer on the GR 916-AL impedance bridge during one of the early flights.

Prop Modulation

The cabin of a Bell 47-G is a bumpy place to bumpy place to bumpy place to bumpy place to bumpy in bridge.

Excelsion-type packing must be used partially to isolate the bridge from shock and vibration. An electrically bumpy effect, prop modulation, is evident when nearing a null on the R-X bridge. Capacitance variations when the wing is fore-off and a towartship (Appendix A, Table 1) are sufficient to modulate the bridge null. Experience and a few

culculations show that errors due to prop modulation will not exceed 5 - 6% in R or X.

Interference

The long vertical wire is an ideal pickup for broadcast and low-frequency radio lange signals. In the Lake Sammamish area, field intensities were in the 10-20 millivolty nater range in the broadcast band. Care must be exercised not to repeat the obvious, but common, error of nulling on broadcast signals rather than signal generator signals. Outside

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Tona, "Study and Investigation of Methods of Dissipation of Static Electricity on Helicopters", U.S. Dept. Commerce No. PB 155-125, Sept. 60

signals are so strong as to cause whistles and squeiks and other super-neterodyne spurious responses. The best solution is to operate as far as possible from BC stations. Highly selective detectors (not VTVMs) are essential to reject strong stitions.

Altitude Effect

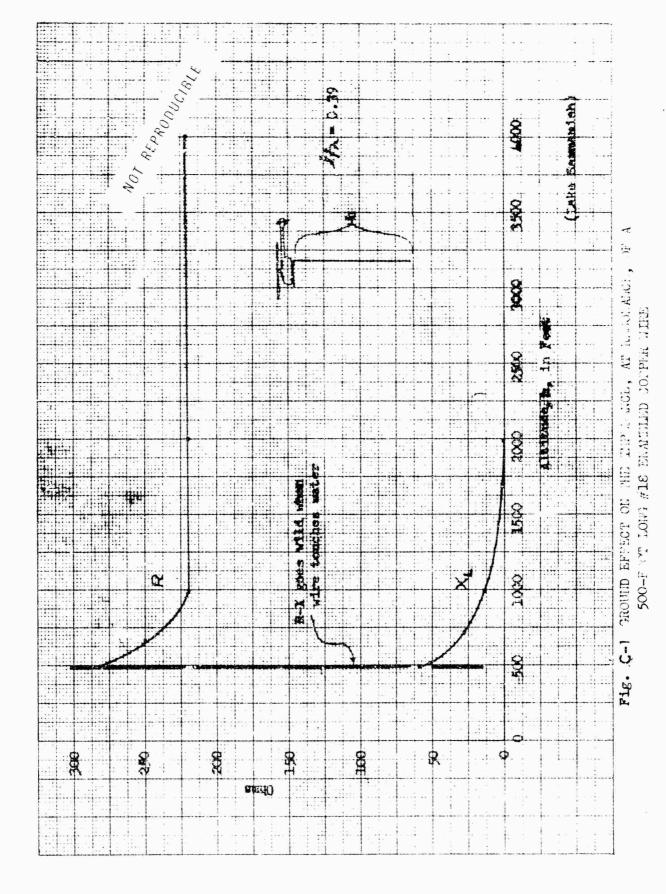
Ground introduces a mirror effect which perturbs the TW impedance. This effect was exploited in the dunking tests over soltwater (see above). The effect of ground on antenna impedance was measured by setting a bridge to null at the "crossover" resonance of a 500-ft wire and then floating downwards from a high altitude. Results are given in Fig. 1. The conductivity of Lake Sammamish and surrounding area is cround 1-4 millionms per meter, leading to an imperfect image in the ground. Over saltwater, a somewhat greater ground effect would be observed. The results of Fig. 1 guarantee, nowever, that the measured data in the main body of this report is substantially free from ground effects.

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Appendix D

IMPEDANCE OF TRAILING WIRE ANTENNA TOWED BY THE KC-135

The nelicopter data can be scaled upward to predict KC-135 TW antenna impedance. The helicopter-to-KC-135 scale factor is $5.5 \pm 20\%$ (Appendix A). The model is then

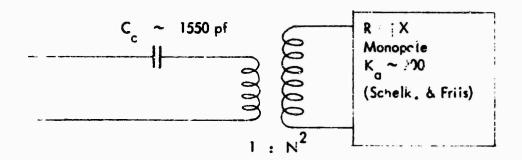


Fig. D-1 KC-135 TV: Impedance Model (from helicopter duta)

The coupling factor, $1/N^2$, will be about the same as in the nelicopter case for corresponding wire lengths; i.e., for a 10,000 foot wire the coupling factor is 0.55. For other lengths the transformer ratio, $1/N^2$, is probably less, but no additional data is available. The spected impedance at 46 kg on a KC-135 towing a two-mile long wire is given in Fig.D-2. The impedance spiral for wires with other efficiencies can be interpolated in Fig.D-2. The effect of base capacitince is not included in Fig.D-2 this must be added in sount to get the antenna impedance at the driving terminals in the aircraft cabin (Appendix E).

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Appendix D

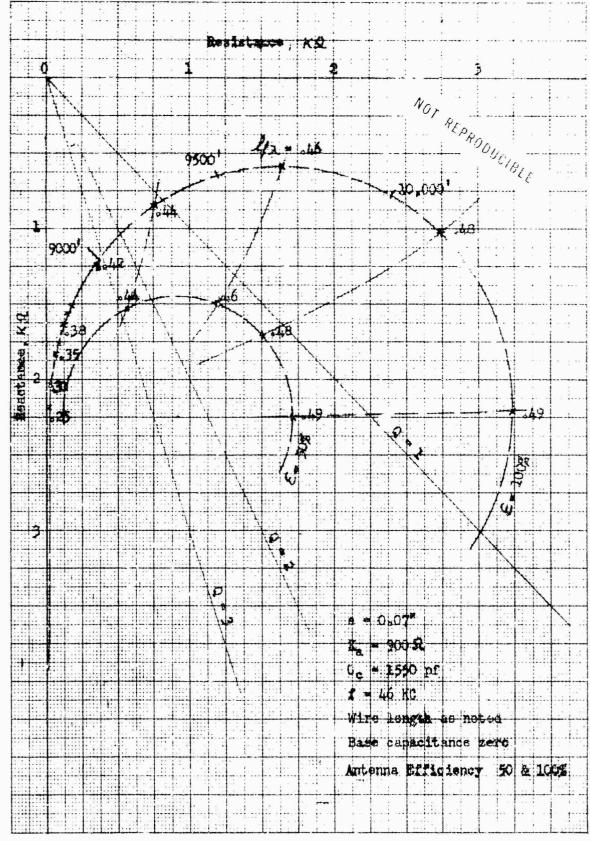


Fig.D-2 EXPLOTED HERE WILL OF A AUTLOS TOHERO A TOHERO WILL WILL WILL A

Appendix E

BASE CAPACITANCE

In practice, the antenno cannot conveniently be fed at the idealized terminals of Fig. 8. The transmitter/timer hust be connected to the antenna via a lead-in. This lead-in introduces base capacitance which raises the Q of the antenna, and may also be the weak point from the standpoint of voltage breakdown.

A semantic di ficulty arises in distinguishing between "antenna" and "lead-in". From one point or view, the "antenna" may be everything autside the pressure-hull of the aircraft. From the point or view or our model for the asymmetric dipole, the "antenna" carries current away from the aircraft, ie., there is no current in the aircraft bucking or parallel to the antenna current (Fig. 1E). The lead-in has mirror currents in the metal walls of the trankline or aircraft skin, the antenna does not. The dividing point, P, is the point

beyond which the currents flow radially from the aircraft. Inboard of point P, the lead-in capacitance is a function of lead-in length and proximity to around, and depends on the detailed geometry of the leadin. In the helicopter tests, ase capacitance out to the point P was carefully minimized to be less than 10 pf. Write this is a design problem beyond the scope of the present report, a few remarks are in order with particular reference to the KC-135 airplane.

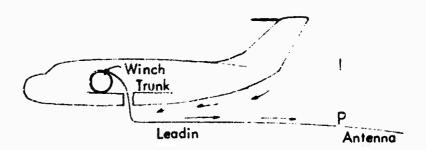


Fig. 1E

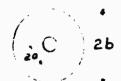
- (1) Base capacitance is, in general, "bad" and should be minimized as it raises the Q and lowers the bandwidth of the antenna.
- (2) The capacitance or lead-in outside the plane can be minimized by using small conductors well away room ground. Small conductors reduce the antenna breakdown voltage and are thus not desirable. But increased spacing to ground is desirable on both counts.
- (3) Lead-in capacitance can be estimated as follows, for the KC-135.
 - (a) Winch Capacitance: Not amenable to direct calculation out depends on the dielectric used to insulate the wind, and the number of turns left on the winch. Several hundred feet left on a winch would give a capacitance of 200-500 pf. For fixed frequency operation the fixed-length plus hylon lanyard scheme used on the helicopter has much to commend it.

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Appendix E FACE 57



$$C_{g} = \frac{2 \pi \epsilon_{o}}{L_{c} b/a}$$



For a 0.1-inch diameter wire in a 3-inch tube,

$$C_y = 5 \text{ pf/-t}$$

 $C_{\ell} = \frac{2 \pi \epsilon_{0}}{\cosh^{-1} \frac{h}{a}}$

(c) Wire to suselage capacitance:
The capacitance of a thin
wire above a flat or nearly
flat ground is,



which for
$$\frac{h}{a} \gg 1$$
 vecomes

$$C_{\ell} = \frac{24}{\log_{10} \frac{2h}{a}}$$
 pi/meter

For a 0.1-inch diameter wire 24 inches from the fuselage, the capacitance is,

$$C_f \sim 3 \, pf/ft$$

In a "typical" KC-135 installation the total bise capacitance would be roughly

$$C_{\text{bise}} = C_{\text{winch}}$$
 C_{trunk} $C_{\text{lead-in}}$

$$= 300 \text{ pf} \qquad 5 \frac{\text{pf}}{\text{ft}} \times 10 \text{ ft} - 3 \text{ pf/ft} \times 50 \text{ ft}$$

$$= 500 \text{ pf}$$

NOT REPRODUCIBLE

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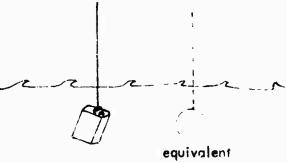
2-700C

NOT REPRODUCIBLE

GROUND RESISTANCE OF A DUNKED WIRE

For certain mobile experiments or for LF com funications, a long wire may be towed with the lower end immersed in water. It is of interest to determine the base resistance and ground losses associated with this sort of grounding arrangement.

A gallon can has been used successfully, as a 'ground connection' for dunking wires. A shiny can is poked with holes and wired to the lower end of the trailing-wire antenna. The can is easily visible several thousand feet below the helicopter with the naked eye, but disappears when submerged. The contact or spreading resistance in water can be computed by use of a capacitance—resistance analogy.



equivalent sphere r = 10 cm

The capacitance of a gallon can in free space is that of an equivalent sphere of equivalent radius $r \sim 10$ cm given by

$$C = 4 \pi e r$$
 (MKS)

By analogy, (the current lines map onto the electric field lines)—the spreading conductance, $G_{\rm c}$, of the same sphere in a medium of conductivity, σ , is

$$G_{s} = 4 \pi G r \qquad (MKS) \qquad (2)$$

In a conducting half-space, with the can near the interface, the conductance is halved, whence

$$R_s = \frac{1}{2\pi G^* f} \qquad \text{onms} \tag{3}$$

In fresh-water lakes, typical conductivities are in the range 1-10 millimho/meter.

Taking for the conductivity

$$G' = 5 \cdot 10^{-3}$$

$$R_s = \frac{1000}{\pi}$$
onms

\$40 5% L F3

The spreading resistance will be about the same at If, where the skin depth in water is much greater than the dimensions of the can. The efficiency of the dunked wire can be computed approximately from the spreading resistance $R_{\rm g}$, and the radiation resistance, thus

$$= \frac{R_{c}}{R_{c} + R_{s}}$$
 (4)

For a quarter-wave dunked monopole

$$R_a = 35 \text{ ohms}$$

and the efficiency becomes

$$\epsilon = 10\%$$

For saltwater, G = 5 mho/meter, and the spreading resistance is

$$R_s = \frac{1}{\pi}$$
 ohms

The efficiency is then

¥1,

$$\epsilon = 99\%$$

In the saltwater case, wire ohmic losses will ordinarily be controlling.